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By

PROFESSOR DAVID EUGENE SMITH

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IN HONOR OF
PROFESSOR DAVID EUGENE SMITH

In February of this year Professor David Eugene Smith retired from active service at Teachers College. At this time his students and colleagues arranged for presenting to the College a portrait of Professor Smith and, at a dinner in his honor, they endeavored to express to him their appreciation of the uniqueness of his work and their regard for him as a friend and teacher.

Doctor Smith's influence on the teaching of mathematics is such that it seems appropriate that the interpretations of his contributions as given on these occasions be made known. In addition, this number of the MATHEMATICS TEACHER contains two articles by Doctor Smith that have not hitherto been generally accessible.

THE EDITORS.

DAVID EUGENE SMITH
Professor of Mathematics, 1901—1926
Professor Emeritus, 1926

From an oil painting by Leo Mielziner, presented to Teachers College by Friends of Professor Smith, April 27, 1926

DINNER IN HONOR OF PROFESSOR DAVID EUGENE SMITH

Introductory Remarks by Professor Clifford B. Upton, the Toastmaster

Friends, we have come here tonight to show the love of the pupil for his master and to voice our appreciation of the work of a great teacher and a noted scholar. But as we start to measure our appreciation we encounter many obstacles. In the first place, it is hard to find an adequate unit of measure; in the second place, it is difficult to fix the boundaries of the subject we are measuring. We start to measure David Eugene Smith, the teacher, but before we finish we are measuring Dr. Smith, the scholar, and suddenly he merges into Smith, the historian—and then he becomes mathematician, and then in turn author, critic, traveller, diplomat, and finally, distinguished representative of American education in some notable International Congress in Europe. Depending upon our point of view his genius changes like the colors of a chameleon. We soon realize that Dr. Smith isn't just one David Eugene Smith, but a dozen of them—he's a superman, or better, a multi-man.

I might add that chronologically the first edition of Dr. Smith is David Eugene Smith, Esq., attorney-at-law. It may be news to many of you that Dr. Smith began his career as a lawyer and is still a member of the New York Bar with the legal right to practice law in this State. Very few of you know Smith the lawyer, and I am going to tell you a little about him. First let me say that he was an unusually skillful one as I know from practical experience, having employed him as legal adviser when, as a young man, I and my roommate found ourselves in trouble with our landlady.

We both had just come to New York, some twenty-five years ago, and had rented a room under certain assurances from the landlady. After occupying the room two days we found that matters had been misrepresented to us by the owner. We explained this as courteously as possible to our landlady, but she turned a deaf ear and told us that we were there to stay; that if we moved she would attach our baggage; and report us to the President of the University. Not knowing the ways of New

York, we decided to consult our good friend Lawyer Smith and, accordingly, called upon him that evening about eight o'clock. His advice was immediate and clean-cut. He told us that we were under no obligation to remain, and advised us to leave and let an action be brought against us if desired. He reminded us that if we could once get out, that would settle it. Following his advice we moved at midnight. The moving was done in homeopathic doses and required some twenty trips with a small suitcase, one of us carrying the bagful of things to our new room a block up the street, while the other stood guard lest we disturb the slumbers of the mistress of that household. The next morning we called upon the landlady and offered her our key but she wouldn't accept it. In fact during the conversation she stood at the extreme end of a long hallway, with us at the other end, so that we couldn't even toss it to her. What an embarrassing situation—we were out of the room but we couldn't get rid of the key! We were sure that Lawyer Smith had miscalculated in his strategy this time. So we called on him again, hoping that he would tell us the magic words that would soothe the heart of an irate landlady—but not so—there was no balm in his prescription—but a lot of practical action. He reminded us from the fullness of his experience that he had never known any woman to refuse a registered letter,—that all women were expecting legacies through the mail from some rich uncle. So he told us to put the key in a letter—before witnesses,—to register that letter, and to demand a registry receipt. We followed instructions, the postman delivered the letter, the landlady signed the card on the dotted line, and we were free once again. That is what I call efficient legal service. I might add that I have never yet received a bill, either for the rent or for those services,—and I now hope that I never shall—because whatever the sum was then, today it must be four times as much because any sum at 6% compound interest would quadruple in that time.

Though Dr. Smith started his career as a lawyer he did not continue his practice very long. Maybe it was because most of his clients were low in funds like myself and my roommate, but I imagine he preferred the life of a student of history. Accordingly he selected as one of his fields the history of mathematics, and in that he has made part of his reputation as a scholar. Just how well he succeeded in this can best be told by Professor Lao

G. Simons, who took her doctor's degree under his guidance and who has had the privilege of working under his direction for many years. I am glad to introduce Dr. Lao G. Simons, Professor of Mathematics at Hunter College.

PROFESSOR SIMONS'S ADDRESS

Mr. Toastmaster, Dr. Smith, members of the David Eugene Smith Fellowship:—

It is my privilege to bring before you one great phase of the activity of the many-sided man whom we honor tonight as our instructor, exemplar, and inspirer.

Do you know what it is to be possessed by a problem, to have within yourself some urge that keeps you at it every waking moment, that makes you alert to every sign pointing the way to its solution; to be gripped by a piece of work so that you cannot let it alone, and to go on with deep joy to its accomplishment?

Dr. Smith has been so possessed these many years in his search for truth in the field of the history of mathematics and the history of the teaching of mathematics.

This search has led him to the ends of the earth whence he has returned with rich additions to knowledge in his chosen field. He has, for instance, contributed to opening up the work of a hitherto unknown great Hindu mathematician; he has shown that China and the Far East deserve a position in the development of mathematics not before credited to them; he has made much more widely known Omar Khayyam, the Algebraist. These three examples constituted just that random sampling which is characteristic of a vast number of cases.

As a result of Dr. Smith's own researches, there have been published, according to a conservative estimate, some three hundred books, articles and reviews in English, French, Italian, German, and Spanish, and perhaps, other languages.

Among the books may be named *The History of Modern Mathematics; The Sumario Compendioso of Brother Juan Diez; Computing Jetons; Rara Arithmetica; History of Japanese Mathematics;* and his great *History of Mathematics* in two volumes, the work of thirty-five years of research.

Among the articles are found that delightful guide to the lover of history, *Historical-Mathematical Paris;* that description of a treasure in the British Museum, *A Greek Multiplication*

Table; that facsimile and commentary, *The First Printed Arithmetic, Treviso;* that "beautiful essay," *Ten Great Epochs in the History of Mathematics.* One knows not what to choose, nor where to place writings of such universal appeal and influence as *Mathematics and Poetry,* and *Religio Mathematici.*

One direct outcome of this outstanding work in the history of mathematics has been the enrichment of the curriculum of many colleges, north and south, east and west, in this country, through the introduction of courses in that subject and the inclusion of historical topics in methods courses in schools of education.

Another outcome has been The History of Science Society with its five hundred thirty-five members, international in character, and including in its membership many great men of the world of science. The inception and organization of this society must be attributed directly to Dr. Smith.

This connoisseur in the history of mathematics is constantly finding treasures in the most unlikely places, such as his discovery of an American Algebra notebook, written in 1739, in a museum housed in the old village jail in York Village, Maine. And that discovery led one of his students to a thrilling adventure in the history of mathematics, an adventure in which he shared as completely as though it were his own.

The ability to do research work for one's self is a gift; but there is here added the ability to awaken in others the love for research, and to show the keenest appreciation of results, small in number and value compared with his own outstanding discoveries. Dr. Smith's students would learn from him his great secret in this respect. That secret lies deep in his own personality, and "Personality" as Havelock Ellis puts it, "is something which cannot be sought. It is a radiance that is diffused spontaneously."

Here, then, we have:

The great research worker,
The maker of research workers,
The influence in the extension of knowledge
through college courses and learned societies,
The great personality.

Toastmaster (introducing Dr. Plimpton): One of the greatest pleasures of which I know is to spend other people's money.

We have with us this evening one who has often given that pleasure to Professor Smith by permitting him to buy for the Plimpton Library any rare manuscript in which he was interested.

As you all know, Dr. George A. Plimpton has assembled the largest collection of early textbooks to be found anywhere in the world, a collection which he generously places at the disposal of Teachers College students who are doing research work in the history of mathematics.

Most of us in this room have spent one or more delightful evenings in Dr. Plimpton's home, hearing him describe his rare books and enjoying his hospitality. In fact, he might well be counted an honorary member of the mathematics staff of Teachers College. He also has the satisfaction and the distinction, as a member of the firm of Ginn & Company, of being the publisher of all of Professor Smith's numerous textbooks. I am very glad that Dr. Plimpton has consented to speak to us again this evening.

MR. PLIMPTON'S ADDRESS

Mr. Toastmaster, Professor Smith, Ladies and Gentlemen:—

It was more than twenty-five years ago that I first had the pleasure of entertaining David Eugene Smith at my home and of taking him into my library of old textbooks. The collection of mathematical texts interested him to such an extent that he said: "My place is in New York City, where I can be near these books." So you see I claim some credit for his presence here tonight.

I would like to say a word about the first arithmetic printed in the English language, and also about the latest one.

Robert Recorde brought out the first important arithmetic printed in the English language about 1542. He did it with fear and trembling, and on the title page he wrote:

"To please, or displease, sure I am
For not of one sort is every man.
To please the best sort would I fain,
The forward displease shall I 'certain.
Yet wish I well, though not with hope,
All ears or mouth to please, or stop."

This book was in use for over one hundred and fifty years, the last edition being used in 1699.

When Professor Smith issued the latest arithmetic printed in English, his knowledge of the subject and the need for the books were such that, unlike Robert Recorde, he had no fear as to the result.

Men have their influence upon education in divers ways. Some assist in progress through their work in the laboratories, some in their efforts on the lecture platform, some in the administrative field, and some in the preparation of textbooks for the schools. It is difficult to say in which of these various lines the greatest influence for good can be exerted; but I wish to say a word upon the influence of Professor Smith as a writer, a subject upon which even his friends often know but little; and never, I imagine, from him.

The demand for his mathematical textbooks has been so great within a comparatively short time that I can say that probably one out of every three of the children in the schools of this country has studied one or more of his works. I cannot tell you the precise number of his books that have been printed, but it may interest you to know that if they were piled one on top of the other, the column would be many times as high as Mount Everest, and that if the leaves were torn out of these books and placed end to end, they would reach several times around the world at the equator.

If Recorde's Arithmetic was in use for one hundred and fifty years, who can measure the influence of Professor Smith's books within the next two hundred years?

Not only have the books met the demands of this country, but they have found a sale wherever the English language is spoken. In addition to this fact, certain of his works have been translated into Japanese, Chinese, Russian, Spanish, Turkish, and Portuguese, or adapted in part in these languages.

The authorship of such a series of books would satisfy the ambitions of most men, but not so in the case of Professor Smith, who has become one of the great authorities on the history of mathematics. He is a collector of autographs and medals of mathematicians, as well as of books, of Oriental manuscripts, and mechanical devices relating to mathematics in its various phases,

and his collections have been of great value to his thousands of students.

With regard not only to my collection of mathematical books but to my other textbooks as well, he has been my unfailing adviser. The success of my collection of early textbooks is due in no small measure to his knowledge and inspiration. Professor Smith is now Emeritus-Professor at Columbia University, but I want to assure him, and you all, that he is not "emeritus" so far as the Plimpton Library is concerned, but he is the Librarian for all time.

Toastmaster (introducing Superintendent W. E. Stark): All teachers of secondary mathematics are much indebted to Superintendent William E. Stark for a series of articles he wrote in 1910 entitled "Measuring Instruments of Long Ago." Those articles did much to put real vitality into the teaching of geometry and have already obtained a permanent place in the literature of the teaching of that subject. At the time he wrote these articles, Mr. Stark was Principal of the Ethical Culture High School, of New York City, and also a member of one of Dr. Smith's advanced classes. He is now Superintendent of Schools at Stamford, Connecticut. I welcome Superintendent Stark here this evening, who will speak of Professor Smith's influence in raising the level of mathematics teaching in elementary and secondary schools.

Mr. Stark's Address

Mr. Toastmaster, Professor Smith, and Friends:—

Those of us who were born in the almost forgotten era of piety remember a story of the downfall of a giant at the hands of a stripling wielding a sling. The youngster's name was David. Verily history repeats itself. We have been watching the assaults of a modern David upon a giant far bigger and tougher than Goliath.

The terror of the Bible story wore armor, but it left a little opening just where a lucky shot would do the most good. Our hero has had no such advantage. The giant upon whom he has been operating had developed the toughest kind of shell from top to toe—a sort of concrete overcoat, made of a mixture of equal parts of tradition, prejudice, and smug self-satisfaction.

David the Wise has made no wild shots, but has used scientific methods. He realized that the monster could be a mighty useful member of society if he could be limbered up and headed in the right direction.

Now to educate a giant is much like educating a child. First, you must understand him. So David began by looking into his heredity, sparing no effort to trace the mathematics family back to its ultimate source. Then he studied the giant's childhood, his playthings, his later development, and everything that would help him to learn how the fellow got that way.

He discovered the main trouble—the creature never took a bath and never changed his clothes. On the rare occasions when he acquired a new garment he just put it on outside the old layers. The burs and the barnacles which he happened to pick up stuck on just as hard as the things worth keeping.

It became evident that an operation was necessary. Economy called for a major operation—like Goliath's,—but when you set out to cut up a giant with a shell like this fellow's—well, don't try to finish the job in one afternoon.

So David began digging a bit here and cutting a bit there and he kept it up. He has already removed several yards of vermiform appendix, which served no purpose except to cause pain. The giant is looking very much better. He has a lot more life and energy. He is much more human—if you know what we mean. He seems interested in real everyday life, instead of merely posing in a museum. School children are actually beginning to like him. Best of all, he has got a new look in his eye. He has stopped gazing backward over his shoulder and keeping his feet carefully in a deep rut. He seems to have an idea where he is going.

There is no logical reason why I should be allowed to speak on this occasion. I have forgotten most of the mathematics which I used to know; and for the last fifteen years, I have been so busy fighting other giants that I have not been able to keep close touch with developments in this field. Nevertheless, I jumped at the chance to say my little word. Like everyone else who has ever worked with David Eugene Smith, I am ready to pop up and cheer whenever his name is mentioned.

Perhaps I may act as the spokesman for the great army of men and women in American schools who are not mathematicians, but

who really want our boys and girls to get the best possible chance of development. We owe a very great debt to this leader in education and many of us appreciate it more fully than we can express.

I cannot evaluate Dr. Smith's influence upon the elementary and secondary schools of the country. I doubt if anyone can do so. But I do know that his influence has been very great. I know that we are indebted to him for ideas which have eliminated a good deal of the waste involved in the traditional courses and methods. I know that his books and the co-operative studies which he has led have spread better ideals and better methods of teaching far and wide. I know that the teachers whom he has trained have gone out into the schools with a vision of educational opportunity, a grasp of educational problems, and an ability to think which are bearing fruit a hundred fold.

Anyone who has had the responsibility of engaging and supervising teachers in a public school system can appreciate what has been going on in the mathematics department of Teachers College He knows that every second teacher used to regard herself as a specialist in mathematics and pointed with pride to her ability to drill a class for routine examinations. He knows that a good many administrative officers used to think that anyone could teach mathematics. There are some of that kind still, but not nearly so many as there used to be. We can now get professional teachers of mathematics—professional in the finest sense—whenever the politicians permit us to employ them.

May I add one personal word? My own work with Dr. Smith was brief and was done long ago. There is no reason why he should remember me, but I shall always remember him as one of the most inspiring teachers whom I have ever known.

He gave me one of the most delightful vacations of my life when he allowed me to spend the summer with his wonderful old books and manuscripts. He gave, in himself, a splendid example of a thorough scholar, a great teacher, and a most lovable man.

Toastmaster (introducing Miss Charlotte Huber): Several times during the past few years Professor Smith has lectured before a general assembly of the pupils in the Horace Mann High School. On one occasion he gave a remarkable Thanksgiving talk and at another time a unique lecture on old mathematical

books, during which, to excite the pupils' curiosity, he drew from his many pockets, one by one, at least thirty rare books and manuscripts, the combined bulk of which was almost sufficient to fill the well-known five-foot shelf. Just how Professor Smith was ever able to walk onto that platform, loaded down with that small library, still remains a mystery to this day. Recently I asked Professor Smith to give those two lectures again for us tonight but he didn't seem to feel it quite appropriate, and so one of the pupils of the Horace Mann School for Girls, who heard both talks, has kindly consented to impersonate him and to repeat those lectures as she and some of her teachers recall them, or think that they recall them, or profess that they think so. To save time she will combine both talks in one, taking the liberty to mix them to suit her taste, and to distort history all that she may wish. It is a real pleasure to announce Miss Charlotte Huber in the role of David Eugene Smith, and to assure you that she will repeat with absolute inaccuracy everything that he never said—or part of it.

IMPERSONATION BY MISS HUBER
DRESSED TO REPRESENT DR. SMITH

Mr. Toastmaster, Professor Smith, Ladies and Gentlemen:—

What in the world have you got to be thankful for? You have to get up in the morning before you want to, you have to go to bed at night before you want to. You are ordered to eat spinach and to drink water. You have to go to school, some of you have to study mathematics out of my books. What have you got to be thankful for, anyway? You even have to come today to hear me talk about *books*.

I wish I could take the time to make you interested in books,—not mine, but those of others. Here is a book about the size of your hand. It was written long, long ago, so long ago that even the wisest students in Teachers College cannot tell you the day and the hour. You can see from the characters inscribed on this tablet the probable origin of *triangular numbers*, and possibly of the *Rule of Three*.

One day I walked into a little shop in India and found one of the *Nine Sections* of the *Ahmes Papyrus*. I was exceedingly glad to get it. It is worth far more than its own weight in gold and for this reason it is known as the *Golden Section*. The reddish

color of the characters suggests that this was the first use of *cardinal numbers.*

The Chou-Peï-Suan-King is the music book from which Pythagoras taught. I think that I must have been Pythagoras in some other incarnation, or at least I may have been his valet and carried his suitcase, and thus learned his teachings of *harmonic* proportion, a method of *scales,* a book on *chords,* and even an *opera mathematica.*

Rara Ethica is a manuscript I bought rather late. One of the volumes I keep locked up in the safe in my office because there are parts of it I don't want other people to see. It deals with Manners and Morals—vulgar fractions, rule of faults, and cross multiplication. The second volume deals with more estimable subjects such as perfect and amicable numbers and *divine* proportion.

The Margarita Philosophica concerns you but might not interest you. You will let me speak to your teachers a moment. This is one of the first books on methods of teaching. We find a discussion of the so-called modern *project* method (here labelled the method of projections), the *galley* method, the *scratch* method, and the method of *exhaustion.* Obviously there is nothing new under the sun.

Long, long ago there was written a pamphlet called *Aljabr wal Muqubalah* by the *Witch of Agnesi.* The translation of the title of this monograph is *Instructions in the Knowledge of All Dark Things,* and its contents are graphic representations of *magic* squares.

The book known as *De Arte Naturae,* or *The Art of Nature,* is really a *Sumario Compendioso* of such works as the *Elements* of Euclid, *Earth* Measurement, *Ground* of Artes, and the *Sand* Reckoner. Eratosthenes, after inventing a *sieve* for sifting sands, wrote the latter book.

The Reviso Arithmetic published in 1650 based on the principles of Pacioli's Psychology was written to delight children and to satisfy the general educator of that day. The author has cast out nines, eliminated unknowns, cancelled all factors, and reduced everything to lowest terms. Even the signs are minus.

In the Pellos arithmetic we first begin to see the point in mathematics.

Here I show you "The Ladies' Diary," by Lilāvati, in which she goes off on a tangent to an osculating curve and talks about male and female numbers, partnership, and the eternal triangle.

I also bring before you the "Algebrista y Sangrados," by Fibonacci, the famous first book on "mixed" mathematics, a fusion of mathematics and anatomy, in which the author discusses Napier's bones, the faces of parallelepipeds, the arms of angles, the legs of triangles, the feet of perpendiculars.

Now the last book I have to show you is the Computus Anianus, a non-Euclidean device (displaying a telephone book) quicker than the slide rule, and good for any number of places. Just call the number to get the answer.

Just remember that there is nothing original in what I have said to you The wonder is that there is anything in it at all. If I were to write this speech I think I should entitle it *Puns Asinorum*. I do not think that is stretching the facts.

It has taken me somewhat less than ten times ten years to learn what I know, and you see I can tell it to you in somewhat less than ten times ten minutes.

At least you have one reason to be thankful—for now—I—am —done.

Toastmaster (introducing Mrs. Pilliod): Some eight years ago, during one of our Summer Sessions at Teachers College, I gave the Rogers test of mathematical ability to some four hundred students who were attending our classes in mathematics. I was much interested to find that a young woman received the highest score in that test. Such a performance on the part of this young woman so attracted my attention that I induced her to remain with us the following year as a teacher in our Horace Mann High School. During the several years she was there she continued to attract attention; in fact, she attracted so much attention of one Mr. Pilliod that she deserted both the school and her fellow teachers in mathematics to become mistress of her own household. She still retains her interest in mathematics, however, and is full of appreciation of the work she had with Dr. Smith. I am very glad to present the one whose name appeared on my class roll as Mary Alice Phillips, but who is now known as Mrs. James Pilliod, who will speak of Professor Smith's influence on the class-room teacher.

Mrs. Pilliod's Address

Mr. Toastmaster, Dr. Smith, Fellow Students and Teachers:—

The educational atmosphere of today is most interesting and invigorating to the lover of mathematics. There is a spirit of clamor and strife directed against this ancient branch of knowledge. We have all felt it, been bewildered by it, perhaps been tempted to stray from our original loyalty to the science. Then out of the turmoil we have each of us at one time or another during the past years stepped into the calm and serenity of Room 212, Teachers College. There we have been reassured that progress does not mean complete denial of the past, but rather the constructive use of it, and that love of mathematics does not imply retrogression but rather progression, and we have gone back to our various classrooms refreshed and inspired to new efforts.

The Master Mind of that quiet room so gently set us thinking along new lines that there was no feeling of confusion or dissension. We might have been antagonized by a didactic statement that apartment house fractions must go out of the curriculum. Instead, a quiet question led us to make the discovery for ourselves. To our surprise we found that these and other familiar highways along which we had driven so unsuspectingly, were in reality blind alleys. We had the pleasure of discovering the equally unsuspected utility of other subject matter and, delighting ourselves in this discovery, we took the good news back and allowed our students to experience the same pleasure.

Even in the dark ages of the past when I was in the latter grades of the elementary school a bit of algebra and something called geometry crept into our horizon for a few weeks. I remember having a kindly feeling toward the latter subject because of a prize won by drawing a large number of neat lines through a point. I presume that I thus discovered a fundamental postulate of geometry, but my teacher did not dream of leading us to associate the subject with the form of a snowflake, the comparative sizes of spherical and hemispherical portions of ice cream in an ice cream cone, or the location of hidden treasure. Neither was algebra connected with such everyday material as a formula for finding one's way about New York City.

We however have been so much more fortunate than that teacher. We have taken away from our work at Teachers College a conception of humanized mathematics which meets the need of the child of today. We have been brought to see that mathematics should be approached from its practical side, from its application to problems within the scope of the child's interests, all this leading to problems which may possibly reach beyond that field but which are of such importance in the development and maintenance of modern civilization as to acquire a social significance in the pupil's cultural background. We have been given the broader view of mathematics which enables us to gather among its admirers in our classes all the different mental types. Its utility appeals to the practical mind; its historical significance attracts those interested in history; while the imaginative are stimulated by the outlook it gives into the vastness of the universe and the mysteries of its control.

Nowhere except in that quiet room could we have had our eyes opened to the close union between mathematics and music, mathematics and architecture, mathematics and poetry, mathematics and religion. Nowhere could we have so regained our faith in our subject, not only in its practical aspect, but in its cultural importance. A subtle but profound influence toward a respect for scholarship made us feel that, for the future of education and hence of the race, delight in pure reason (the "magic of the mind") should not be entirely neglected for the merely practical. Each summer has come a reminder of this in the delightful monographs—*Poetry and Mathematics, Historical-Mathematical Paris, Mathematica Gothica*—by which we have been kept in touch with this broadening influence and our outlook has been still further widened and deepened.

We have thus been inspired to test our own wings. We knew that kindly encouragement would help us as we floundered in new fields and tried our individual experiments. I am certain that those who are making outstanding advances in the problem of greatest importance today—the enrichment and expansion of the subject matter and courses of the Senior High School—would frankly admit who it was that inspired their efforts. A few years ago I encountered the college professor who taught me mathematics. "Since you left college," he said, "we have grown very brave. We are trying a six weeks course of calculus

on the freshmen," and I answered, "Do you call that brave? Even the sophomores in our Horace Mann High School are thoroughly enjoying their six weeks unit in the same subject." We all know the source of the courage and inspiration necessary to make this important experiment in secondary education.

Not only was our delight in our subject matter increased by the contact with the genius of that quiet room, but we were perhaps unconsciously molded into new methods of teaching by the force of a perfect example. We saw what it meant to follow a teacher who was never too busy to help his students in their individual difficulties. We saw how we ourselves were inspired to work because our honest efforts were met with sympathy and because nothing shoddy could stand the searchlight of that keenly critical mind. We saw what force and impetus a single quiet question may give and we learned that guidance may be so subtly given that no hint of it comes to mar the pleasurable sensation of original discovery and self-imposed activity.

And so our minds were awakened to new possibilities in mathematics and to better methods of imparting this broader aspect of our subject by the force of an example of unfailing courtesy, kindly tact, patience illumined by a sense of humor, and a keen reason which pierced to the root of the question and eliminated everything but honest, clear-cut thought; and, for this influence toward wider horizons we have to thank Professor Smith for being as he is, a scholar and a gentleman. Perhaps I may be permitted to close my own remarks by reading both the Latin and the translation of a tribute paid to Professor Smith by one of his colleagues, the sentiments being so much in accord with all who have known him as a teacher:

DAVIDIS EUGENIUS SMITH
doctor noster et magister

Et qui doctor! quam acutus, quam eruditus, quam artis suae peritus! Cum eum laborantem spectabamus, facile perspicere poteramus, qualis is esse deberet, qui rem mathematicam docere vellet; cum autem docentem eum videbamus, tum denique intellegere poteramus, quo modo hoc faciendum esset. Quid dicam! Et auctor et magister, imitandus est et non imitabilis.

DAVID EUGENE SMITH
Our teacher and master

> And what a teacher! How brilliant, how learned, how skilled in his art! When we beheld him in his study we could readily see what a teacher of mathematics should be; but when we saw him teach, then could we understand how this teaching was to be performed. What more shall I say? Both as author and teacher, he stands as one to be imitated, and yet as inimitable.

Toastmaster (introducing Professor Keyser): It is our good fortune that Dr. Cassius J. Keyser, Adrain Professor of Mathematics at Columbia University, and colleague of Professor Smith, is with us this evening. At my suggestion, made less than an hour ago when I found that Dr. Keyser was able to be here, he has kindly consented to speak to us. It is with the greatest pleasure that I introduce Professor Keyser.

ADDRESS BY PROFESSOR KEYSER

Mr. Toastmaster and Friends:—

I have been warned not to speak over three minutes, and I was not commanded to say anything important. I shall not speak long, but I may say one or two words that will take the form of congratulation. As I conceive it, the congratulations will be two in number, at least of two kinds. First, the congratulation of Professor Smith himself on the great field of his mathematical activity, "The History of Mathematics," and I congratulate him upon the choice of that field, because in my opinion, it is the field that best reveals what is most precious and most significant in our common humanity. I mean that composite ability of man, of all human beings, to make the achievements of past generations the means of achieving still greater things; of making science breed science, philosophy breed philosophy, and art breed art. This composite ability of human beings it is that has made our civilization. Without it our past civilization could not have existed, and without it our present civilization would not advance. And this composite ability to make the achievements of the past the means of still greater achievement, is, I say, the most significant, the most precious characteristic of human

beings as human beings. And where is it manifest? It is manifest in all of the fields, certainly in all of the great fields, of human activity.

Where is it most clearly manifest? Unless I am mistaken, it is most clearly manifest in the field to which Professor Smith has given the major part of his life—the history of mathematics. For there the involved method of the past in the evolution of science is not only manifest as it is in other departments of human activity, but it stands out in its nakedness, and so it is no wonder that Professor Smith is a humanitarian. That leads me to my second congratulation.

I congratulate all of you and myself upon having been the pupils or the colleagues, or both pupils and colleagues, of such a man as we have learned to know him to be. Not all of us have had the pleasure of being pupils of Professor Smith, but all of us have had lessons from him. His presence has been a living lesson to us in gentleness and in kindness. His presence has been a living lesson in the elevation of spirit above the sordid things of the world. It has been a living lesson of catholicity of appreciation. It has been a living lesson in magnanimity of spirit. It has been a living lesson in that quality of judgment in art which critics call good taste. It has been a living lesson in loyalty—loyalty to pupils, loyalty to colleagues, loyalty to his great subject.

And now I have another congratulation. Congratulation of all of you and of myself, and that is that although we shall not be henceforth officially pupils or colleagues of Professor Smith, we shall yet be, as I understand it, and as I hope it will be true, for many years his neighbors.

Toastmaster (introducing Professor Smith): We have heard this evening of the many things which Professor Smith has done to improve the teaching of mathematics in the schools of this country. There is still another contribution, however, which I shall be glad to add to the list. It was Professor Smith who created the first genuinely professional course for the training of teachers of secondary mathematics that was given in this country. To explain this statement I must go back some thirty years to the time when education as a professional subject was found only occasionally in our colleges and universities. There

were, however, at that time, a few courses of collegiate grade on the teaching of the various academic subjects, such as mathematics, English, and history. So far as mathematics is concerned, these courses were largely reviews of the traditional subject matter as taught in the high school, and nothing appeared in the discussions which dealt with the principles of selecting appropriate subject matter, with improved methods of presenting the various topics, or with the study of the world literature bearing on the vitalization of the work in our field.

In other words, the modern type of teachers' course in algebra or geometry did not then exist save in Professor Smith's classroom at the Michigan State Normal College. He offered at that time, however, what I can truly say was the first model of the teachers' course in secondary mathematics as we have it today, and when I look back upon that course I realize how little we have done to make any substantial improvement upon it. We have added a few pedagogical phrases that were not then in use, and we now include a few special topics, such as tests and measurements, which did not then exist; but, in the great essentials, he offered at that time as he has since one of the most inspiring courses on the teaching of mathematics that has ever been given in this country. We therefore owe to Professor Smith the distinction of having created, and since that time of having constantly set the pace for, the highest type of professional training of the teacher of mathematics.

I am sure we will all regard it as a most fitting climax to this evening's program if we may now have the pleasure of hearing from the one in whose honor this dinner is held.

Dr. David Eugene Smith, master, teacher, beloved friend, and inspiring colleague, we welcome the opportunity to be with you tonight and hope that you will say a few words to us.

PROFESSOR SMITH

Mr. Toastmaster and Friends, Colleagues, Students:—

I have listened with very great pleasure to the descriptions of someone whom I do not know. It is like hearing voices far off. I cannot imagine the man of whom you are speaking. For myself I might say a few words, but not for the man who has been described in such glowing terms.

It is an interesting thing to search for words with which to express clearly our feelings. There are emotions that you can never describe in any language. The best words that we find are inadequate. "Appreciation" is one of these words; but how feeble it sounds, and how difficult it is of definition. The dictionary fails to tell its full significance, and all that we can say is that appreciation is simply appreciation. I should like to express my appreciation for this kindness you have shown me, and I should like to express for my dear wife the appreciation which she felt today when she received the flowers which you so thoughtfully sent her, and to express adequately her regret that she could not be here tonight; but to do all this is to fail to express all that the word signifies to me. I recognize, too, in this gathering an outpouring of affection and a spirit of what, in the Teutonic tongue, is known as "Gemüthlichkeit." The former term we cannot define, and the latter one we cannot translate. If I were to attempt to express the meaning of the three words, "appreciation," "affection," and "Gemütlichkeit," I think I should say they signify what we have here this evening. It is a feeling of thankfulness, of good friendship, of family ties, of similar interests. Perhaps it may not seem to be affected if I say that the words refer to an emotion prompted by the words "love of our friends"—at any rate this is what I feel, although I cannot adequately express my thoughts. Perhaps I have said enough to assure you of my appreciation of this evidence of your affection and of the "Gemütlichkeit" that characterizes this occasion.

I have been forty-two years at the work of teaching, and it seems no more than forty-two weeks. A good old friend of mine came to me tonight and said, "Is this Professor Smith?" He didn't know me, and I looked at him and did not know him. It was Superintendent Patrick, of New Jersey, and he was one of my pupils in the early days when I took my seat on the rostrum at the end of the faculty row. I had taken up the work rather by chance, simply to "help out" for a time in a normal school that needed a teacher, and I agreed to stay for six months only. Well, it has been longer, but it doesn't seem so. Good old Bede, "Bæda Venerabilis," when he began one of his classic books more than a thousand years ago, wrote, as was the custom then, on the first vellum page the words, "Incipit feliciter"—"It beginneth happily"; and so forty-two years ago, after the first

day's work, I wrote on the first line of the day's record these same words—"Incipit feliciter," for I felt that a new volume had begun. And when at the end of his vellum tome the good old "Venerable Bede" reached the last sheet, he dipped his faithful quill in the inkhorn and wrote "Explicit feliciter"—"It endeth happily." So tonight I dip my quill in the ink and write his words, "Explicit feliciter"—and am made the happier because you are here and have done this for me. I have always been impressed, also, by a certain passage in the works of the old monk of Jarrow. "I have spent my whole life," he says, "in the same monastery; and while attentive to the rule of my order and the service of the Church, *semper aut discere, aut docere, aut scribere dulce habui*"—"my constant pleasure lay in learning or teaching, or writing." So I may say tonight, "For twenty-five years I have lived in this college, and while I have attended faculty meetings and worked (under mental protest) upon committees, 'always to learn, or to teach, or to write, these things have been my delight.'" If I were to pick out an epitaph to sometime be carved upon my tombstone, I think I should pick out these same words—"Always to learn, or to teach, or to write—these things have been my delight."

It is natural, at a time like this, that a speaker should indulge in reminiscences; but being natural, I shall not do it—at least to any extent. I wish simply to say that during these years there has been a revolution that we cannot possibly appreciate in this generation in which we are living. Only the Future can see its significance. In the government of the world, in religion, in the habits of men, and in education as well, there has been a revolution the like of which the world had never seen before. This revolution has shown itself in mathematics as in all other lines of human interest. In my boyhood mathematics was taught to only a relatively small number of young people. Only a relatively small number of pupils completed their high school, and a much smaller number went to the colleges. In the small city in which I lived, not more than a half dozen had ever gone to college before my time. Now they go annually in groups of considerable size. We all know what the revolution has done in secondary mathematics. The work is not taught more thoroughly, but it is much better taught. While our students do not know many of the details of mathematics that were taught a half

century ago, many more students have a working knowledge of the subject. Real graduate mathematics was just beginning to be known at that time. The great mathematical centers which we have in our country today did not then exist. In our secondary schools we have greatly broadened the courses, and as a result of that movement and of the impetus given to advanced studies in our universities we have developed a quality of work that in the early day was wholly unknown.

It has been pleasant to stand on the bank and to see the progress of the stream. It has been still more pleasant to have had even a small part in the movement. This part, for myself, has not been what my friends have claimed tonight. It has been a far more humble one. I may perhaps describe it as that of a long-robed priest who simply lights, in some little chapel, seven small candles. And to these seven candles I may give fanciful names, just to increase your interest in my humble task.

I may speak of the first as the *lampas utilitatis,* because we cannot convey mathematics to the great mass of people unless we first dwell upon the utility of the subject and imagine what would happen to the world if every trace of mathematics and of mathematical knowledge were blotted out tonight.

The second candle has been the *lampas decoris,* the lamp of beauty; because if we are to teach mathematics at all, real success is not possible unless we know that the subject is beautiful as well as useful. Mere utility of the moment without any feeling of beauty, becomes a hopeless bit of drudgery, a condition which leads to stagnation.

The third has been the *lampas imaginationis,* which has always seemed to me especially appropriate in referring to a medieval cathedral in which we set our lamps, and which seems equally so in respect to our chosen science; for what would mathematics have amounted to without the imagination of its devotees—its giants and their followers? There was never a discovery made without the urge of imagination—of imagination which broke the roadway through the forest in order that cold logic might follow.

The fourth candle has been the *lampas poesis,* the lamp of poetry; because if one does not feel the poetry in mathematics, one may as well cease teaching the science. What, after all, is mathematics but the poetry of the mind, and what is poetry but the mathematics of the heart?

The fifth of the candles that we all seek to light is the *lampas mysteriae*. This it is that reveals to us one of the great charms of the science—that in working in the domain of mathematics, you are surrounded by clouds, and success drives back these clouds a little way, and a discovery is made; then someone makes another discovery and drives them back a little more; and at rare intervals in time a Newton comes and drives them back, what seems a long, long way—and still there is the surrounding mist of mystery. It is a great experience, this piercing the clouds; but try as we may, there is still the mist about us.

The next lamp has been and is the *lampas infinitatis*, the lamp of the infinite. A writer not long ago, in a verse which appeared in one of our magazines, spoke of mathematics as the science which lassos the flying stars. It means much to have played even a little around the outskirts of "the science venerable," to have seen how it reveals something of our own position on the great macrocosmos, and to see what an infinitesimal thing we seem when we look at ourselves in the light that mathematics sheds upon this cosmos. The other day a mathematician took a certain measurement, and it was by no means one of the greatest of our time. He found that one of the other universes about us was six quintillion miles away, one million light years. The greatest speed that we can obtain mechanically by any present means is the speed of a rifle bullet, which may go half a mile in a second —a velocity so great that we can hardly imagine its possibility. If we ask how long would it take the rifle bullet to reach that other universe, even our very elementary mathematics gives us the answer—it would take three hundred eighty billion years. Truly it is "the science that lassos the stars."

And the seventh of the candles that this humble priest in the little chapel of the great cathedral has essayed to light is the "*lampas religionis.*" We may wonder if such a candle burns and sheds its light, but I have an idea that we all feel that, while a mathematician may not necessarily be a very religious man, on the other hand no man can appreciate religion to the full unless he has to assist him some knowledge of the great field which mathematics opens to his vision. Mathematics may not make any man more religious, but if he is religiously inclined it makes him see the grandeur of religion as nothing else can.

The forty-two years have been short. It is a comedy that it is so short, and that is the tragedy of the matter. This very fact makes us see the truth of our old Persian friend, known all over Persia as the great algebraist of his country, and all over the West as the great poet of his people. He describes the camps of his native Khorassán, and portrays the old slave who strikes the tents preparatory to the arrival of the newcomers of the day, and does so in these well-known words:

"'Tis but a Tent where takes his one day's rest
A Sultán to the realm of Death addresst;
The Sultán rises, and the dark Ferrásh
Strikes, and prepares it for another Guest."

Knowing this instinct of the race—that it is the fate of life itself—it is the the fate of life itself—it is with some pleasure, and certainly not with pain, that I place my foot in the stirrup of my waiting camel, swinging myself to my place upon his back, and turn his supercilious face ominously toward the west, and start out upon my trek across the desert toward the Garden of Allah. As I go slowly along, now and then glancing ahead, I see other camels which have already arrived at the oasis. I see my good old friend Dr. Sachs, and Dr. Woodhull, and Professors Farnsworth and Nutting, all of whom are resting in the shadow of the palms. And I see, coming along behind me, my good friends, Professors Keyser, Hawkes, Fite, Upton, and Reeve, who are all here tonight, and so I am not at all lonely as I travel slowly over the Sahara. Moreover, I know that among you who are with me at this feast there are others who will become *professores emeriti*, and these will pass along the same route. And when the camels reach the spot and kneel beside the sweet waters, you will raise your arm and will say, after the manner of the East, "*Salam alai Kum*" and I will reply "*Alai Kum salam*," "Peace upon you!" and "Upon you peace!" And you will find me waiting, as old Omar said, with "a book of verses underneath the bough, a jug of wine, a loaf of bread," and thou wilt join me there in what we hope may prove to be the placid, the pleasant, and the peaceful oasis of the *professores emeriti*.

THE CALL OF MATHEMATICS

AN ESSAY BY PROFESSOR SMITH

Why is Mathematics Studied? Ever since man came to think in the abstract, to think of the number two as distinct from two objects, to create for himself units of measure that possess some approach to uniformity, to think of time, and to be aware of such concepts as lines and angles, mathematics has been an object of his study and the basis of most of the natural sciences of antiquity, becoming the very essence of all science of the present day. In every generation men have arisen to question its value; when these men have died, their protests have died with them; and in our time other men have arisen to ask the same question, and with them the protests will die as with their ancestors.

The question is, however, a fair one; it would be asked by the mathematician if it were not asked by those to whom the science was so poorly taught, or so unsuccessfully, as to warp their judgments and to encourage their element of destructiveness. In the one case the query would be raised in a sympathetic frame of mind; in the other case, in a hostile spirit; but in any case it is legitimate and it demands a frank answer.

The Utility of Mathematics. In an age that seems to be more utilitarian than its immediate predecessors, but which probably is not so, it is natural, in weighing the value of mathematics, to speak first of its manifold uses. We study mathematics, then, because it is one of a small group of subjects—like reading, history, and geography—that are linked up with a large number of the branches of human knowledge. No one can be happy as a member of the human family who does not know something of the history of the race, something of the earth on which this race exists, something of letters, something of the arts, and something of what we pedantically call "the quantitative side of human life." Of the necessity for knowing number relations there can be no question, but fifty years ago one might well have asked, for example, why the girl should study algebra. Today, a person would sadly feel his ignorance, or her ignorance, if he or she had to look with lack-lustre eyes upon a simple formula such as may be found in the popular scientific journals, in an

everyday article on astronomy, in a boy's manual on the airplane or radio, in a book on nursing, or in any one of hundreds of articles in our popular encyclopedias, and had to confess ignorance of its significance. These needs come not only within the purview of the boy; they are even more apparent in the case of the girl,—she who is to have the direction of the education of the generation next to come upon the stage of action. Each must know the shorthand of the formula and the meaning of a simple graph, of a simple equation, and of a negative number, or else must feel the stigma of ignorance of the common things about which the educated world talks and reads.

If we are skeptical as to the reach of mathematics in the world in which we live, let us consider the fact that, aside from the propagation of the race, the most important thing in this world is education,—in which term we may include the training of the soul as well as the training of the hand and brain, and the training for the eight hours of daily leisure as well as for the eight hours of toil. Let us then take the science of education as a norm for measurement and imagine, if we can, that by some mighty cataclysm there should be wiped off the face of the earth tonight every book on mathematics, every mathematical symbol of any kind, every written page or printed sheet upon which a trace of mathematics appears, and every machine for computing or recording numbers; and then let us do the same for every piece of printing or writing that has to do with the science of education. What would happen? There is an ancient adage that it is an ill wind that blows no good, and some good would unquestionably come to the world were this done. For one thing, modern warfare would cease, since shells of the right kind could not be sent to the proper guns, and the range finders would fail to operate. For another thing we should doubtless have more attention given to real teaching because there would be some lessening of experiment, valuable though this may be in a fair per cent of cases, particularly those in which the approximate measure of pupils' abilities is concerned, and exceedingly valuable as it will probably be in time to come. The actual teaching in the schools would go along about as usual; very likely, however, with less friction for a time. But how about life beyond our scholastic walls? Every mill in the whole world would slow down and every large concern would close until it could replace

its accounts, its statistical material, its formulas for work, its measures, its tables, and its computing machinery. Every ship on the seven seas would be stricken with blindness and would wallow helpless, awaiting a chance view of a haven of refuge or else the starvation of its human burden. Not a rivet would be driven in one of the gigantic buildings of our growing cities, because the steel girders would have lost their numbers; Wall Street would close its portals; the engineering world would awaken tomorrow morning to a living death; the mines would shut down, labor would have no employment, and trade would relapse into the condition of barter as in the days of savagery. It is a picture that is so ridiculous that we smile at its very impossibility. But it is a real picture, ridiculous though it seems,—a picture of the world sending forth the emergency call for help, a hurry call for the return and the aid of mathematics.

All this would be agreed to by every person of intelligence; but it is asserted, and with reason, that this is simply an argument in favor of mathematics for mathematicians, not of mathematics for the masses. The question therefore naturally arises as to how much of the science may be said to constitute a fair minimum for the fairly well-educated citizen.

Minimum Essentials. When we speak of the minimum essentials of any subject, we mean the essentials necessary for the average, well-educated man or woman, somewhat mythical though this average individual may be. The moron may need nothing that the science has to offer, but the average man in the street is neither a moron nor a genius. What does mathematics have to offer him?

It is not necessary to specify all the details that he can use. He must know the common operations with integers, but he will probably never need such work in fractions as illustrated by the case of $3\frac{13}{16} \div 2\frac{5}{64}$; he will need to know how to find $12\frac{1}{2}\%$ of \$250, but probably not how to find the number of which $0.5\frac{5}{9}$ is $33\frac{1}{3}\%$, simple as the problem is; he will need to know certain parts of economics and certain business customs which find place in arithmetic, such as the investing of his savings and the meaning of a bill of goods, but he will probably never meet with the necessity of finding the present worth at compound interest of a life annuity beginning at a future time; he will need to know

how to evaluate such simple formulas as are found in books on the radio, on nursing, and on the elementary trades, and such as are needed in small commercial transactions, but he will probably not have to deduce any of these formulas for himself; he will need to know how to interpret a graph, and even how to make simple graphs relating to his business, but it is not probable that he will have to determine the nature of a very complicated statistical graph nor to draw one; he may have to know most of the elementary rules or formulas used in the common measurements of familiar objects, but his chance of having to find the area of an ellipse is very remote; and he will probably neither have to demonstrate a proposition in geometry in any experience of an ordinary business life, nor have to solve a quadratic equation. If all there is in mathematics were to be limited to the immediate demands of the man in the street, it would not be necessary to go farther than the elementary school, and the same may be said of reading, geography, such sciences as biology and mechanics, and of the subjects of learning in general.

A Step in Advance. Man has not, however, been content with the merest minimum, whether with respect to a place of rest, to his clothing, to his food, or to his thoughts. If he had been, the world would never have emerged from barbarism. Man seeks for general knowledge of the great things of life; he is not content to think of the stars as lights hung in a vast inverted bowl, or to be ignorant of the general laws that make possible the telephone, the flight of an airplane across the continent, the sailing in safety of a steel ship weighing a hundred million pounds, or the locating of a point on the earth's surface by its latitude and longitude. Such knowledge invites mankind to come to it, and mankind replies. It is not the cry of physical hunger that moves us to action; it is the call of the intellect of man. The former must be heard; the latter will be; and the educator who listens only to the call of the body is certain to fail in his efforts to stimulate youth to its best endeavors.

Comparisons. Questions arise with respect to all the other common branches of knowledge similar to those that arise with respect to mathematics. Why should any one care to know where Paris is unless he is going there or is engaged in commerce?

Why, indeed, should anyone know that the earth is round? No one uses this fact in painting a barn or in milking a cow or in running a taxicab. Why should any one know what Shakespeare wrote, or whether he wrote anything? Are we any better able to prophesy as to a baseball score by having knowledge of even a single one of the great pieces of our literature? Are not those feeble educators correct who say that the best literature for the school is the daily newspaper? Certain it is that this is what the American reads the most assiduously,—especially the sporting and the financial pages and the advertisements. Why should children and youth study biology, or general science, or American history, or history of any kind? It is of no financial value that we know the date of the discovery of America, or even of the declaration of American independence, or that America was ever discovered or ever severed the bonds that tied it to a foreign government. We can live with the narrow vision of the nomad of the desert if we choose, and may even prosper in this world's goods. We may join that group which calls only for the statistics and the art and letters of the present, resting content with complete oblivion for the achievements of the past in education, in architecture, in government, and in literature; or we may claim our inheritance in the glories of the past and thus receive new inspiration in attacking the problems of the future. It is for this reason, among others, that we pay at least some attention to history, and it is for similar reasons that we seek to impart a modicum of information about the various other leading branches of human effort and of human interest.

Influence of Geometry upon Related Subjects. Mathematics makes other calls upon our attention, and one of them is that it has an indirect influence upon all our varied interests. If, for example, the knowledge of how to arrange a logical proof in geometry can be made of no value to us in other fields in which deductive logic can be applied; if the perfection of geometry does not give us an ideal of perfection that helps us elsewhere in our intellectual life; if the succinctness of statement of a geometric proof does not set a norm for statements in non-mathematical lines; if the contact with absolute truth does not have its influence upon the souls of us; if the very style of reasoning does not transfer so as to help the jurist, the physician, the salesman, the

publicist, and the educator; if the habit of rigorous thinking, which usually begins first in demonstrative geometry, is not a valuable habit elsewhere; if a love for beauty cannot be cultivated in geometry so as to carry over to stimulate a love for beauty in architecture,—then let us drop demonstrative geometry from our required courses. Merely from the standpoint of actual measurement, aside from the transferable power it gives us for independent investigation and the pleasure it gives us in the study of its truths, geometry is not worth the time and energy it takes.

It is not a mere desire to call up the aid of a potent name, or to make a fetish of a national hero that leads to a question from Lincoln's story of himself; but the words are those of a man of much broader intellect than those of the critics of the influence of geometry, and of one whose powers of introspection were fully commensurate with his powers of statesmanship. His statement as to geometry is as follows:

"In the course of my law reading I constantly came upon the word *demonstrate*. I thought at first that I understood its meaning, but soon became satisfied that I did not. * * * I consulted all the dictionaries and works of reference I could find. * * * At last I said, 'Lincoln, you can never make a lawyer if you do not understand what *demonstrate* means,' and I left my situation in Springfield, went home to my father's house and stayed there till I could give any proposition in the six books of Euclid at sight. I then found out what *demonstrate* means and went back to my law studies."

It is this transfer of the power derived from geometry to the study of related subjects that is one of the reasons for teaching the science, but it is by no means the only one. We teach it because it is one of the eternal verities,—one of those disciplines that arouse youth to a contemplation of the truths that endure. Did you ever think how we might proceed to make an attempt to communicate with Mars by signals?—how we would place enormous searchlights to form some picture on the Sahara, some picture that the Martian might be certain is not a mere accidental arrangement, or an accidental series of flashes, and yet something that he would understand? It could not be a representation of a living thing; for if Mars has life it is not probable that its forms are shaped like those of Earth. It could not be words

in letters which have been transmitted to us through the ages, nor could it be the numerals which had their visible source ages ago in the royal decrees of India. No, it would be none of these; perhaps the most hopeful symbol we could give to attract the attention of a world much older than our own and probably more refined, would be the figure of the theorem of Phythagoras, the squares on the three sides of a right triangle. We may smile at the idea,—but it would be difficult to think of a better symbol; and the reason is that here is one of the verities of the universe. Before Mars was, or the Earth, or the sun, and long after each has ceased to exist; there and here and in the most remote stellar regions,—the square on the hypotenuse was, and is, and ever shall be, on our hypotheses of space, equivalent to the sum of the squares on the sides. All our little theories of life, all our childish speculations as to death, all our trivial bickerings of the schools,—all these are but vanishing motes in the sunbeam compared with the double eternity, past and future, of such a truth as this.

Summary of the Mathematics Required. In general, then, it may be said that the well-informed citizen needs to know the ordinary arithmetic of daily life; the algebra of the formula, graph, negative number, and the simplest form of an equation; the intuitive geometry of common forms, of position, and of simple mensuration; the meaning and significance of indirect measurement; and the meaning of a geometric proof, this being the ground covered in our junior high schools. Beyond this the work may safely be reserved as an elective subject for those who show ability in mathematics or special need for some of its branches. Such an arrangement of the curriculum would be better for the pupil than the more traditional one, and much better for those possessed of some native ability in the science. To the select group the call of mathematics is something beyond the physical; it is the call of the soul, precisely as in the case of music, of painting, and of other fine arts, or of science, or of letters. It is this call that must be answered if mathematics, the fine arts, the sciences, and letters are to advance and make the world a better place in which each succeeding generation is to play its part in the progress of the race.

Do the Schools Teach Mathematics at its Best? But do we recognize all this in our teaching of the subject? As a rule, no, and it must be confessed that we recognize but little of it in our teaching of anything. We fail to recognize adequately the story of the development of the rights and duties of the individual in our teaching of history; we commonly fail to recognize the noble chants of Gregory in the teaching of music; we fail to recognize the sonorous language of the Athenian and the real significance of the oration on the crown in our passing reference to Greek literature; and we fail to grasp the overwhelming significance of education when we spend our weary hours in reports, and in questionnaires, in tests, and in regulating the bells of our class rooms.

Frankly, we cannot sanction, as perfect, anything that is in this world of ours, and so we cannot fully sanction the present teaching of mathematics or of anything else. Surely our governments fail to meet our ideals; and surely our Christianity, our Judaism, our Mohammedanism, our Buddhism,—these failed to ward off the display of savagery that staggered the world in the great war of 1914-1918. Our divorce courts, our children's courts, our brothels, our crime, our poverty,—all these cry to heaven of the failure of our social system. There exists nothing in this world that cannot be made better, and so let it be said at once that mathematics is poorly taught, but no more poorly than pedagogy; that it contains a mass of material of questionable value to young people, but no more than sociology; that its subject matter can undoubtedly be much better arranged, but the same can be said with even greater force of psychology; that its beauties are not made adequately manifest, but that the same can be said of music and *belles lettres*. Let us, therefore, assume no brief for the perfection of the teaching of mathematics as it stands; but if we had to take a brief for this subject or for the philosophy of education, or for practical arts, or for courses in social betterment, it is probable that each of us would take that of mathematics without the slightest hesitancy, and would do this because it has a more scientific and logical basis, and because it has had a far longer time in which its method of presentation could develop towards perfection.

When we think of the record of failure in the proper presentation of such subjects as religion, and art, and educational theory,

and mathematics, we may be tempted to lose heart,—but a moment of reflection should assure us that we are constantly tending towards the good. Someone may tell us that not 0.1 per cent of pupils attain a mark of 100 in algebra, and that 30 per cent fail altogether; but when we reflect that not 0.001 per cent of humanity attains a mark of perfect in matrimony, and that many more than 30 per cent fail altogether, we do not at once begin a crusade to abolish marriage. Nor is this analogy either farfetched or ridiculous. If any one says that we must have matrimony or else the race will perish, the reply is that the statement is not scientifically accurate, nor has it been scientifically demonstrated that it is necessary that the race should not perish now instead of some millions of years hence, as it apparently must. If any one says that we force the girl to take algebra, but that she is free to accept or reject matrimony as she chooses, then we may safely deny the assertion, for millions more of girls are forced, in one way or another, to marry than are forced to take any particular subject beyond the most elementary work of the schools. If any one says that we should improve the conditions that make matrimony so often a failure, then we may reply that many and perhaps all are seeking to do this very thing,—the woman becoming the intellectual companion of the man, the man and woman tending to accept the same social standards, the division of labor becoming more scientifically agreed upon, and the woman assuming her share of the burdens of the state; but in rejoicing in this, we must also say that our failure to progress more rapidly does not mean that we should abolish matrimony, even though this has substantially been suggested in our present generation and by a considerable body of human beings. Similarly, many of us are doing our best to improve the teaching of algebra, and geometry, and all the other mathematical disciplines, and our slow progress does not mean that we should advocate the closing of the doors of the subject to the youth of our day, although this too has been suggested by another considerable body of people.

The call of mathematics is, then, to our physical well-being, and this is always recognized; but it is also to our spiritual well-being, and this we must not fail to recognize if our labors are not to be in vain.

THE POETRY OF MATHEMATICS

AN ESSAY BY PROFESSOR SMITH

Poetry in Mathematics. Weierstrasse, than whom few men of his calling were better able to speak with authority, once remarked that "a mathematician who is not somewhat of a poet will never be a perfect mathematician"; and Thoreau, who is not often suspected of possessing the mathematical tastes that really were his, went even farther when he wrote:

"We have heard much about the poetry of mathematics, but very little of it has yet been sung. The ancients had a juster notion of their poetic value than we. The most distinct and beautiful statements of any truth must take at last the mathematical form."

Even more clearly and with more feeling did Thoreau's friend, "the Sage of Concord," express the idea:

"We do not listen with the best regard to the verses of a man who is only a poet, nor to his problems if he is only an algebraist; but if a man is at once acquainted with the geometric foundation of things and with their festal splendor, his poetry is exact and his arithmetic musical."

Such expressions of the sentiments of poets and of mathematicians might easily be found in the literatures of all lands and all eras, but to the mathematician, to the successful teacher, and to the poet of real imagination no repetition of the thought, even when clothed in new expression, seems necessary. It is more worth the while to mention a few of the elements of "the science venerable" to which the term "poetical" particularly applies and to intimate the opportunities that naturally arise for leading the younger disciples of mathesis to see the vision.

Mathematics in Poetry. Just as mathematics, to the mathematician and to one who teaches the science, is filled with poetry, so poetry welcomes mathematics to herself, arranging her message in meter and her sonnets with mathematical precision. In like manner she reveals, as the mathematician does, the beauties

of symmetry, and she commonly designates her rhythmic lines as "numbers," as witness the verses with which Longfellow begins his *Psalm of Life*,—

"Tell me not in mournful numbers
Life is but an empty dream,"

an unconscious popular witness to the union of the most ancient branch of science with the most ancient branch of letters. Not content with form alone, poetry continually draws upon the ideas of mathematics in seeking contact with a double infinity,—the infinitely large and the infinitely small, as in the writing of Lucretius and in those of many of lesser fame. It is contact with the infinite that has been the dream of the sage as seer, as poet, and as mathematician since the days when the world was young, and this will endure until the world is old, for it is an instinct of the race, the instinct that separates it from the brute.

Mathematics and Poetry. But it is not mathematics in poetry, or poetry in mathematics that beckons our interest so much as it is mathematics *and* poetry,—the union of the two as equals in the display of the results of man's imagination. Dr. Thomas Hill has expressed the union of mathesis learning and poesy (*poiesis*, creation) in these words:

"The Mathematics are usually considered as being the very antipodes of Poesy. Yet Mathesis and Poesy are of the closest kindred, for they are both works of the imagination. Poesy is a creation, a making, a fiction; and the Mathematics have been called, by an admireer of them, the sublimest and most stupendous of fictions. It is true, they are not only *mathesis*, learning, but *poiesis*, a creation. . . .

"Mathesis and Poetry are . . . the utterance of the same power of imagination, only that in the one case it is addressed to the head, and in the other, to the heart."

Points of Contact. If we seek for points of contact between mathematics and poetry they are easy to find and not difficult to bring into the consciousness of even the humblest learner,—not as a burden but as what the jurists call *obiter dicta*, things said casually.

Voltaire, for example, once remarked, "One merit of poetry few will deny; it says more and in fewer words than prose." This is not a definition of poetry, it is simply a characterization. The sonnet is an example, with its need for embodying a great thought in a limited number of lines. The quatrains of the old Persian algebraist, Omar Khayyam, and those in Tennyson's *In Memoriam* furnish other examples. The little gems of Japanese poetry, the *haiku*, in which some noble thought is expressed in precisely seventeen words, are perhaps the most exquisite specimens of all. This is seen, to take a single example, in Dr. Nitobe's translation of the lament of a mother on the death of her child:

> "How far, today, upon the chase, I wonder,
> Has gone my little hunter of the dragon fly,"—

words of which the full significance can be appreciated only by those who have watched the play of children in the land of happiest childhood, and the philosophic resignation of the people in the midst of great affliction.

But see how equally significant are the words of "the Old Man of Ferney" if we change them to read, "One merit of mathematics few will deny; it says more and in fewer words than any other science." The world knows no such conciseness of expression outside this particular field. Mathematics, indeed, is the very example of brevity, whether it be in the shorthand rule of the circle, $c = \pi d$, or in that fruitful formula of analysis, $e^{i\pi} = -1$,—a formula which fuses together four of the most important concepts of the science,—the logarithmic base, the transcendental ratio π, and the imaginary and negative units.

Poetry and mathematics have this feature then in common,—each says more and in fewer words than any other written forms.

Was Byron thinking of mathematics, or was it of poetry, when he wrote his lines about the magic of the mind?—

> "The power of
> Thought—the magic of the mind."

It might have been either, for where does the seeker after truth feel this "magic of the mind" as he does in reading the adven-

tures of Æneas, of Ulysses, and of Sir Launcelot, unless it be in those symmetric expressions of algebra which are themselves verses and which set forth at its best "the power of thought." It is not a mere caprice of etymology that links the verse of letters with the versed sine (the "turned sine") and the *umbra versa* (the "turned shadow") of the early trigonometry, and with the vertical of geometry. Each is a turn of form and each is a turn of sentiment as well.

To one who cannot appreciate the rhythm of such symmetric expressions as

$$a^2 + 2ab + b^2$$

and

$$a^3 + 3a^2b + 3ab^2 + b^3$$

all suggestions of the poetic form of algebra are meaningless; but to him whose master was a poet such expressions combine perfect symmetry, perfect rhythm, a perfect geometric picture—like the façade of the Parthenon,—and a perfect example of "the power of thought,—the magic of the mind."

The uplift of both poetry and mathematics is due, and probably chiefly due, to the fact that each is seeking for the truth. As Bacon phrases an ancient aphorism, "No pleasure is comparable to the standing upon the vantage ground of truth." We may make a pretense of denial of the sentiment, but we shall search in vain for any pleasure so great. Surely it is not found in the sensual, nor can it be bought by wealth alone, nor does dress or position or travel or leisure or labor bring it in any such degree as "the standing upon the vantage ground of truth." Macaulay sets forth this merit of poetry in these words:

"The merit of poetry, in its wildest forms, still consists in its truth—truth conveyed to the understanding, not directly by words, but circuitously by means of imaginative associations, which serve as conductors."

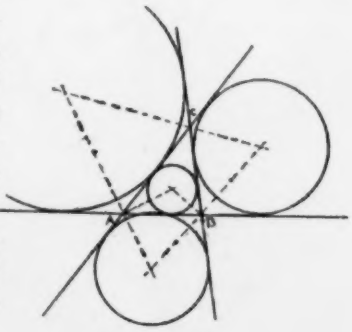
We might easily recast the sentence and say that the merit of mathematics, in all its forms, consists in its truth; truth conveyed to the understanding, not directly by words but by symbols which serve as the world's only universal written language. We may, indeed, think of poetry and mathematics as two conductors, each bearing to our better nature imponderable currents of electrons of truth,—truth which we feebly attempt to express in words but which strengthens our whole inner being.

And yet, with all this inability to express the full lesson that either poetry or mathematics seeks to convey to us, how culpable is the teacher who fails to reveal to his pupils, if only through a casual remark from time to time, the great thought that the merit of mathematics, like that of poetry, in all its forms, consists in its truth! For in all the world, "no pleasure is comparable to standing upon the vantage ground of truth!"

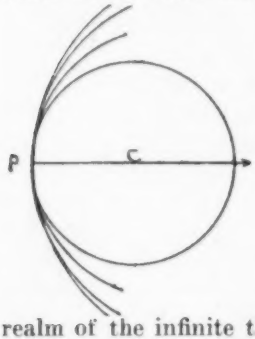
Mathematics and poetry also join in their consistency, a feature seen in the non-clashing of rhythm and of rhyme, of algebraic symbols and their geometric analogues, of the spheroids which make up our solar system, and of the groups that work out like the figures in a kaleidoscope. Mathematics is the science of consistency; it is a picture of the universe; as Plato is said to have expressed the idea, "God eternally geometrizes." Mathematics generalizes with perfect safety, whether in Lineland, in Flatland, in our own space, or in hyperspace, for it is the paramount science of consistency, the science which is valid in the "domain of anyness."

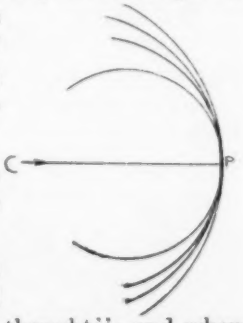
Perhaps the strongest bond of sympathy between mathematics and poetry, however, is the endless invention of each. Dr. Johnson remarked,

"The essence of poetry is invention; such invention as, by producing something unexpected, surprises and delights"; but he might have said the same of mathematics. A student who is asked to construct all possible circles that shall be tangent to the three lines determined by the points A, B, C, and then to consider the nature of these circles when C moves so as to rest on AB and then to a position on the other side of AB, and then still further down "to infinity," and to take various other positions,—this student has, indulged in "such invention as, by producing something unexpected, surprises and delights."



In the same way we enter the realm of invention and surprise when we consider the simple case of a circle generated by the movement of a point P about a point C as center. If we first fix P and let C move to the right, the circle becomes larger, inclosing more and more of space and approaching nearer and nearer a straight line. Not seeking at present to justify the statement, we say that, when C reaches infinity, the circle becomes a straight line; shutting up all on either side; and as the center passes through the realm of the infinite the circle reverses its bend and becomes smaller and smaller as C approaches P from the left. Possibly it was some such vision as this that led Sir Thomas Browne to write:





"Circles and right lines limit and close all bodies, and the *mortal right-lined circle* must conclude and shut up all."

When we say that "poetry" is unfallen speech" we may properly add the dual proposition, "Pure geometry is undebased thought"; and when we say that "architecture is frozen music" we may add that it is also crystallized geometry. There are no cornerstones that mark off the domain of mathematics from the domains of music and poetry and architecture and the fine arts in general, save those imaginary ones which ignorance or indolence of thought have sought to erect. When Bailey, in his *Festus* wrote the lines:

> "Poets are all who love,—who feel great truths
> And tell them,"

he was not thinking merely of those who set forth their thoughts in verse, but of all "who feel great truths, and tell them." And when the elder Disraeli asserted that "philosophy becomes poetry, and science imagination, in the enthusiasm of genius," he was addressing the world of "wisdom-lovers" in general, and in particular those who feel that mathematics may become and does become poetry in the enthusiasm of an inspired and an inspiring teacher.

PRESENTATION TO TEACHERS COLLEGE OF A PORTRAIT OF PROFESSOR SMITH

On April 27 a portrait of Professor Smith was presented to Teachers College by some of his friends. It was painted by Leo Mielziner, an artist well known for his portraits of Presidents Roosevelt, Wilson, and Coolidge, Justices Stone and John Bassett Moore, and other men of prominence in public life. On the occasion of the presentation, addresses were delivered by Professor C. T. McFarlane on behalf of the intramural donors, George A. Plimpton, Esq., on behalf of the extramural donors; Mr. Mielziner, the artist; and Dean Russell, on behalf of the Faculty. The portrait was accepted on behalf of the Trustees by Mr. Macy, President of the Board.

The addresses are as follows:

Professor C. T. McFarlane speaking for the intramural donors.

Mr. Macy, Dean Russell, Ladies and Gentlemen:

It was in February, 1901, that David Eugene Smith became Professor of Mathematics in Teachers College. It was in February, 1926, that he resigned his professorship after 25 years of service. It is interesting to note, in this connection, that during this quarter of a century he was absent from his classsroom but four days, except when away on College business,—a record that is unusual if not unique.

The influence of his teaching has spread as far as the influence of the College itself, as his students have come to him from all parts of our own country and from many and distant lands. His friends, however, feel a greater pride in the fact that for every earnest student who came he had something worth while to give.

In a land and day of educational bolshevism he succeeded in teaching a respect for scholarly methods of study and research, and in pointing out to those who came seeking after truth a clear and sunlit path through the jungle of modern pedagogy. He himself held, and he had no hesitation in teaching, an ideal of mathematics that found a use for number and for measure outside the shop, and we owe him much for what he has shown us of its application to the beautiful and the eternal.

It has been a matter of great satisfaction to his friends "within the walls" to know that the Trustees of the College have set aside

office and library space where students may still, upon occasion, consult with him in regard to questions of research within his field and where they may continue to enjoy the use of the unique mathematical library that, out of his own means, he has brought together.

Appreciative as his friends are of his scholarly genius and his great teaching ability, it is not after all for those qualities alone, or chiefly, that we love him. It is rather because he has been a kindly human brother with us all along the way. It is because in times of need he has always been a wise counsellor and a good friend. Of the depth and sincerity of the friendships that he inspires I, who have known him longer and more intimately than any one else here, except the members of his own family, give today a somewhat unwilling proof. I am a poor speaker upon occasions such as this, and only real friendship brings me here.

A few of Professor Smith's friends within and without the walls had this painting made, so that it might be given to Teachers College. It is our belief that those who have been his students, as they come back to these halls, will be glad to find it here. We believe that his reputation as an author and teacher has contributed to the reputation of the College itself, and that his pictured presence will be of value to the institution even in the years to come when all who knew him personally have passed along the way.

It is with genuine pleasure that, representing over 400 contributors, I now present this picture of Professor Smith to Teachers College.

For the Extramural Donors

DR. GEORGE A. PLIMPTON

Mr. Macy, Dr. Smith, Ladies and Gentlemen: Dr. McFarlane has said that Dr. Smith missed four days from his classes in twenty-five years. If I had known that, I should not have come. I thought him perfect, and now I am disillusioned. Nevertheless, he has his good points, and I wish to say a few words concerning one of them that I have not heard emphasized in the recent addresses and articles relating to his achievements. He is ordinarily thought of as a mathematician, and he has certainly done more than most men to make this science known to young people. He is also thought of as a teacher, and few have

achieved greater success than he in this important field. He has high standing as a teacher of teachers, and thousands in this country can testify to his gifts in this branch of his activities. He is widely known as a historian, an essayist, and a reviewer, and his contributions amply prove the justice of the world's judgment in this respect. But what I wish to mention today is his genius for knowing the child mind. I hold in my hand a copy of his *Number Stories of Long Ago*, and I will read you just a few sentences.

"It is so very, very long ago that not even the wisest men of China can tell the year or the century in which little Ching, the king's eldest son, played in the forests at the foot of Mount Yu, and painted a face on the shell of his biggest turtle, and told the soldier who guarded him what a lot of turtles he had. To be sure, Ching had only three turtles, but he didn't know a word for 'three,' and the soldiers didn't, and not even the King could do more than say, 'Yes, there are a lot of turtles.' "

I have not time to tell you the rest of the story; I simply read these lines to show you a phase of Dr. Smith's activities with which you may not be familiar. This little book has opened the vision of many thousand children to the interesting story of a subject that is often rendered inexcusably dull and uninteresting. It shows the love of the author for children, and I take it that it is this love that has contributed most to the success that he has had in his chosen field.

It has been a great pleasure to his friends "without the walls" to have a share in the presentation of this striking portrait to Teachers College.

Remarks of Mr. Mielziner, the Artist

Gentlemen: After the distinction was conferred upon me by the committee of being selected to paint the portrait of Professor Smith, he called upon me in my studio one day so that we might make each other's acquaintance. There were no casualties on either side. Then he very graciously invited me to visit him in his study on the second story of this building. I was anxious to beard the lion in his den—not exactly to beard but to face the lion, to study his habitat, learn his background and surroundings. You see the portrait painter likes to find the personal background of a new subject. On the walls of that room I found charts,

graphs, and other samples of his profession. There were charts geometric, charts trigonometric, charts algebraic and charts arithmetic. Of course I was interested, but at the same time filled with consternation. Would it be necessary to paint the man in terms of his profession? I had visions of having to buy mathematical implements, and spoiling a nice new canvas with various mathematical things,—"sines," I think you call them, don't you? and cosines, and perhaps tangents and various other things, and then the terrible thought came to me,—would I have to treat my subject as plane or spherical?

Fortunately, however, I was given closer contact with the man, and as I grew to know him, I discovered that although he was four-square and upright, he was not narrow, and his luminous mind was not consecrated entirely to the cold world of science. As I came further to know him, I found it was necessary to mix my paints with love and devotion. One of the greatest privileges of the portrait painter is the opportunity he has of coming into close contact with men and women of distinction, who have found a lasting place in the niches of fame. Of the few who stand out with great distinction, David Eugene Smith is the one I shall always hold not only in great esteem, but in great affection.

Remarks of Dean Russell

Mr. Macy, Professor Smith, Mr. Plimpton, and Students:

This occasion is one of mingled pleasure and pain for me. A pain because I see in it the beginning of the end of an epoch in the history of Teachers College. Twenty-five years ago a few of us, "The Old Guard" we are sometimes called, were faced with the problem of working out ideas and the development of standards for an institution the like of which did not then exist. This required vision, courage, devotion, professional ability and scholarship. Not all of us could qualify in all of these respects, but there was one man in that group who I think would have been named then as eminently qualified in each.

My first touch with Professor Smith was when I heard him deliver an address on the Beauties of Algebra. I did not know until that day that there were any such things, and I listened with surprise and admiration to the man who not only could see the beauties of algebra, but the beauties, the practical uses, and the joys of mathematics in general and of scholarship of every

kind. Those days of beginnings were days of hard work—days when we did not have the easier courage that characterized those who joined the group later on; and if there be something of pain, there is a tremendous amount of pleasure in the memories that we have of the days, and the nights, too, when we were planning and scheming for a future into which we could not see far.

Professor Smith came to us with a mathematical mind, an acute intellect, and a powerful personality. In those intervening years he demonstrated that scholarship ripens under responsibility and opportunity until, as has been said here, his reputation is not confined to our country, but is both national and world wide. That understanding heart of his, as well as a professional fitness, means that he has to his credit in the world today a host of disciples; and if to this you add what comes to the public through his authorship, I think we can safely say that there is no man in this field in his generation that approaches him in extent of influence in our profession. That is a record which comes to but few, very few of us in any field. As an outstanding professional accomplishment, it is almost unique in our experience.

And now, as has been well said by Dr. McFarlane, I fancy that most of us in this college, in thinking of him, pass over his eminence as a scholar, and his outstanding superiority as a teacher and a writer, and take into account primarily those personal qualities which have endeared him to us all. He has the genius for friendship, and I doubt if there is any person, young or old, who have ever come close enough to him to be thought of as a friend, who has ever lost that friendship. These personal qualities and these personal relations make the parting doubly hard; but you know there is, in spite of all this, something of pleasure and satisfaction in that, while we lose the professor, we keep the man. We have him stored away still on the second floor, and I know I am voicing the feeling of every person here when I say that we rejoice in the fact that we can still knock on that door and receive the same kind of welcome as before, and I hope that the years may be many in which we can still enjoy that pleasure.

Among the great friends of Teachers College, who have given service, there is no one who has given us more than this friend. The endowment that he presents, is to use Kipling's phrase, beyond the price of gold, in fact it is an endowment that money can not buy.

For the Trustees

V. Everit Macy, Chairman, Board of Trustees:

Here at Teachers College we appreciate perhaps as nowhere else the fact that great teachers are not made through the mere acquisition of knowledge but by that intangible something we call personality. All the training in the world cannot take the place of the human touch possessed by all great teachers. The world-wide influence of Teachers College has come through the strong characters who have developed the institution and from the fact that their real interest was not in the mere acquiring of information but in the personal contact with their students and in the understanding and inspiration they gave to others. Our leaders have never tried to impose their personality or views, but have known how to strengthen and bring out the best in others. Professor Smith is one of those great teachers whose character and ability have helped create the traditions of Teachers College from an early period. His devotion to his profession, his deep interest in learning and his sympathetic understanding have made him an outstanding figure. The College is fortunate in beginning its Hall of Fame with such portraits as those of Dean Russell and Professor Smith. Others in time will be placed beside them so that each generation can visage the type of men and women who have made the College famous, thus being able to realize the high courage and indestructible imagination of these leaders in education.

In behalf of the Trustees of Teachers College and in the name of the countless students, I accept this portrait of Professor Smith with deep appreciation of his accomplishments as a scholar, his influence as a teacher, and the affection he has inspired in all his associates.

Address of Professor Smith

Mr. Chairman, Mr. Dean, Friends: I wish I could find words with which to express adequately my sincere appreciation of the honor you have done me in presenting this portrait to our College, and in coming here today to give me the pleasure that I now feel. This, however, is impossible. The heart does not communicate through the tongue nor does it receive communications through the ear; it sends its messages not by telegraphy, not by

telephony, but by telepathy. Such a message is now being sent; it is "on the air"; may you "tune in" to receive it.

Words failing me to express my thanks as I should like, let me tell you two brief stories. They are modern stories,—that is, they have no point; they are mathematical stories, circular in form, each ending precisely where it began.

The first is a story of three good friends of mine who dwell in a palace. It is a noble building, nobly placed, and facing the noblest avenue in the world. Once it belonged to a great king, "le roi soleil" he was called; it later belonged to a great emperor, called by some a butcher and by others "the magnificent"; it now belongs to a great people,—great in the arts and great in the sciences. My three friends abide in a long and richly decorated hall, and they always await me there. They know that I shall enter by the door leading to the west from the Salle Carrée and I always find them looking eastward as if to welcome my coming. I walk a short way, and there at the right is the first of these old friends. He seems pleased to see me, as I am to see him, and then we are silent,—that silence which bears witness to true friendship. You may ask, "What is the name of your friend?" I do not know, nor does he know mine. All I know is that the little inscription reads, "Portrait of an unknown man, by Raphael." For four centuries the name and the work of the artist have survived, but the name of the sitter has long since been forgotten.

I walk down the great hall and stop about midway, turning to the left. Every visitor does the same, and bows at the altar of my second friend. I look in her face again, and she smiles,—the smile that has captured the world. What is her name? I cannot tell. The Latin clerks of her day spoke of her as "Mea domina Elizabetha,"—my Lady Elizabeth; the Italians softened it to "Madonna Elizabetta," and the Venetians to "Monna Lisa." In our day we drop all the music of the name and forget its significance and tell how we have seen "the Mona Lisa." The name of the sitter is lost, but the most-admired portrait in the world endures, and with it the name of the greatest genius of his age, Leonardo da Vinci.

I walk down towards the end of the hall and stop at the right. Here my third friend awaits me. Few visitors give him even a

passing glance. These catch the initials of the name, DES, and pass on. But I have known him for half a century and I stand and silently talk with him. His eyes seem to look just over my shoulder; they are the eyes of a dreamer. His mouth,—is it just about to speak, or has it spoken? The pose is characteristic. He seems glad to see me. For three centuries he waited for our language to offer to scholars the work that made modern mathematics possible. He and I have a bond of sympathy that few who pass us by can understand. This is why I stop and spell out his name in full, DESCARTES, while for most of the other visitors to the Louvre the name of Frans Hals, the artist, is all that is noticed.

My second story concerns three other friends. They dwell in a lofty hall. The light struggles in through the *grisaille* windows and the atmosphere is that of a cloister. I say to the attendant, "That is a fine portrait of Barrow; who was the artist?" "I do not know, sir," is the reply; "I only know that Dr. Barrow was a great teacher; he was Newton's teacher, sir." Then I look at the face of my second friend and say: "And that portrait of John Wallis,—that is also a fine piece of work. Can you tell me who painted it?" "No sir," is the reply; "I only know that Dr. Wallis was a very learned man, skilled in mathematics and conversant with the languages of the East as well as with the classics of the West. He was a friend of Newton's, sir." Then I look at the portrait of my third friend, and I say: "That is a noble face; I like it better than the one by Vanderbanck." "Yes," is the reply; "many come here to see Sir Isaac's portrait, but if you should ask who painted it, sir, I could not tell you. Newton brought great honor to this college and this university, sir."

These are the stories, and here is a portrait. Which is to endure, the name of the sitter or the name of the one who moulded this head, seemingly in three dimensions, and projected it upon a plane? You are my friends, my personal friends, and to you I might appeal for friendship's sake. But you love good art, you recognize the artist's skill, you are honest,—and honestly must your vote be cast. Which shall it be, the sitter or the artist?—in Stockton's phrase, "the lady or the tiger?"—and which is the lady and which is the tiger? Here, then, is a dilemma; which of the two lemmas do you choose?

In medieval dialectics the one attacked always sought a *tertium quid,* and there happens in the case before you to be a "third something,"—but you have not seen it. It is carved upon the frame, beyond your present reach of vision. It reads, "Presented by Some of his Friends." May it not come to pass, then, that the future will soon forget the name of the sitter; perhaps even the name of the skillful master of the brush; but that wanderers through these halls will say, "This man had friends; he must have had many friends; and he must have been a friend to many." They may not write him down, like Abu ben Adhem, as one who "loved his fellow men"; but just as "one who loved his friends." And so it may well be that this painting will not stand as a witness to the sitter's worth or to the painter's skill, but as a witness to human friendship. If such shall be the case, then shall I say, in the rhythmical Tuscan tongue, "sono contento."

PROFESSOR SMITH'S LITERARY ACTIVITIES

By JEKUTHIAL GINSBURG

Some time ago the writer made a list, as complete as possible, of Professor Smith's literary productions extending over a period of thirty years. No attempt was made to search out the books, essays, or reviews which he wrote during the first ten years of his teaching, but it is well known that he began his literary activity early. At the request of the editor a brief abstract has been prepared which will serve to give some idea of his productions from and after January 21, 1895, on which date he was thirty-five years of age. No attempt has been made, however, to list his encyclopedia articles appearing in three standard publications and running up into the hundreds, nor teachers' manuals.

Altogether, in this period, he wrote either in whole or in large part, about a hundred books. The separate published volumes number about 130, from which must be taken such as represent the breaking of certain textbooks into two parts. The number of reviews is upwards of 100, and the number of essays and monographs is about 155. Owing to lack of space the names of collaborators on textbooks are, in general, omitted. In some of the earlier books they include Professor W. W. Beman, and in numerous later ones Mr. George Wentworth.

A very condensed list of these publications, with the titles of the reviews and essays omitted, is a sfollows:

1895 1. *Plane and Solid Geometry*, Ginn.

1896 2. *Plane Geometry, Ginn.*

3-6. Reviews. *School Review*, 3:569, 572, 572, 573.

7, 8. Essays. *L'Intermédiaire*, 9:395.

9-11. Reviews. *School Review*, 4:102, 180, 239.

12. Essay. *Educational Review*, 13:348.

13. *History of Modern Mathematics*, Wiley.

1897 14. Book review. *School Review*, 5:184.

15. *Klein's Famous Problems of Geometry*. (Translation.) Ginn.

16. Essay. *Education*, May.

1898 17. Essay. *Bibliotheca Math.*, 1898, p. 13.

1899 18, 19. 2 reviews. *School Review*, 7:45, 46.

1900 20. *Elements of Algebra*, Ginn.
21. *Solid Geometry*, Ginn.
22. *The Teaching of Elementary Mathematics*, Macmillan.
23. *Fink's History of Mathematics.* (Translation.) Open Court.
24. Review. *School Review*, 8:49.
25. Translation. *Open Court*, 14:385.

1901 26. Essay. *L'Enseignement Math.*, 3:157.
27. Review. *School Review*, 9:481.
28. *Geometric Paper Folding.* (After Row.) Ginn.

1902 29. *Academic Algebra*, Ginn.
30, 31. Reviews. *Bulletin Math. Soc.*, 8:353, 9:123.

1903 32. *Higher Arithmetic*, Ginn.
33, 34. Essays. *Proc. N. E. A.*, 555; *T. C. Record*, 4:91.
35, 36. Reviews. *Bulletin Math. Soc.*, 9:218, 376.
37. *Educational Games.* Cincinnati.

1904 38. *Primary Arithmetic*, Ginn.
39. *Intermediate Arithmetic*, Ginn.
40. *Grammar School Arithmetic*, Ginn.
41. *Advanced Arithmetic*, Ginn.
42. *Algebra for Beginners*, Ginn.
43. *Grammar School Algebra*, Ginn.
44. *Practical Arithmetic*, Ginn.
45, 46. Reviews. *Bulletin Math. Soc.*, 10:200, 261.

1905 47. *Handbook to Arithmetic*, Ginn.
48. *The Old and New Arithmetics.* (Pamphlet.) Ginn.
49. *Portfolios of Eminent Mathematicians.* 2 pts. Open Court.
50-52. Essays. *L'Enseign. Math.*, 5:469, etc.
53, 54. Reviews. *Bulletin Math. Soc.*, 11:554; 12:314.

1906 55. *History of Modern Mathematics.* Revised. Wiley.
56. Essay. *Educational Review*, 31:300.
57-60. Reviews. *Bulletin Math. Soc.*, 12:309, 314, 456; 13:139.

1907 61-64. Essays. *Bibliotheca Math.*, 7:375, etc.
65-67. Reviews. *Bulletin Math. Soc.*, 13:302, 392, 506.

1908 68-71. Essays. *Bibliotheca Math.*, *L'Enseign. Math.*, etc.
72. *Rara Arithmetica*, Ginn.
73. *Rara Arithmetica*, 2 vol. ed. de luxe, extra plates.

1909 74. *Complete Arithmetic*, Ginn.
75. *The Teaching of Arithmetic*, Ginn.
76-88. Essays. *Tokyō Butsuri, Monist, Archiv*, etc.
89-93. Reviews. *Bulletin Math. Soc.*, 15:188,190,244,245, 386.

1910 94. *Oral Arithmetic*, Ginn.
95. *Plane Geometry*, Ginn.
96-99. Essays. *Bibliotheca Math.*, 11:79, etc.
100. Review. *Bulletin Math Soc.*, 17:312.

1911 101. *Plane and Solid Geometry*, Ginn.
102. *Plane Geometry*, Ginn.
103. *Solid Geometry*, Ginn.
104. *Vocational Algebra*, Ginn.
105-107. *Arithmetic*, Book I-III, 3 vols., Ginn.
108. Essay. J. W. A. Young, *Monographs*.
109. *The Hindu-Arabic Numerals*. (With L. C. Karpinski.) Ginn.
110-112. Essays. *Tohoku Math. Journ., Amer. Math. Monthly*, etc.
113. Review. *Bulletin Math. Soc.*, 17:484.
114. *The Teaching of Geometry*, Ginn.
115. *Plane and Solid Geometry*. Chinese ed., Shanghai.

1912 116, 117. *Topical Arithmetics*, primary and advanced.
118. *Work and Play with Numbers*, Ginn.
119. *N. Y. State Arithmetic*, years 5-8, Ginn.
120. *The Portrait Medals of Sir Isaac Newton*. Pamphlet.
121-127. Essays. *Jahresbericht, Zeitsch. math. Unter.*, etc.
128. *Bibliography of Teaching of Math.*, 1900-12. (With Goldziher.)
129, 130. Reviews. *Bulletin Math. Soc.*, 19:84, 86.
131. Introduction to Rangacharya's *Mahavīracārya*.

1913 132-134. *Algebras*, Academic, School I, School II, Ginn.
135. Essay. *Isis*, 1:197.
136, 137. Essay. *Math Teacher*, 5:161; separate pamph. revised.
138. Essay. *Educational Review*, 45:1.
139-143. Reviews. *Bulletin Math. Soc.*, 19:246, 248, 249, 310, 367.

1914 144, 145. *Modern Arithmetics*, Pri., Adv., Ginn.
146, 147. *Plane and Spher. Trig., Plane Trig.*, Ginn.
148. *Trigonometric and Logarithmic Tables*, Ginn.
149. *Academic Algebra, with Logarithms*, Ginn.
150. *Plane Geometry*. (With G. St. L. Carson.) London, Ginn.
151. *Plane Geometry*, Pt. I. (See 150.)
152-154. New Jersey ed. of *Arithmetics*, I, II,-III, Ginn.
155. *Elements of Algebra*. (With Carson.) London, Ginn.
156-160. Essays. *Napier Memorial Vol., Roger Bacon Essays*, etc.
161. *History of Japanese Mathematics*. (With Y. Mikami.)
162-167. Reviews. *Bulletin Math. Soc.*, 21: 130, 138, etc.

1915 168-170. *Modern Arithmetics*, Va. edn., 3 books, Ginn.
171. *Modern Advanced Arithmetic*, Canadian ed., Ginn.
172-175. *Essentials of Arithmetic*, Pri., Int., Adv., Ginn.
176. *Plane Geometry*, Pt. II. (See 150.)
177. *Elements of Algebra*, Pt. II. (See 155.)
178-181. Essays. *L'Enseign. Math.*, 17:53, and pamphlets.
182-189. Reviews. *Science*, 41:793; 42:128; *Bulletin Math. Soc.*, etc.
190. *Budget of Paradoxes*. 2 vol. ed. of De Morgan. Open Court.
191, 192. *Spanish editions of two geometries*. Ginn.

1916 193-200. *N. Y. City Arithmetics*. 8 vols. for 8 grades. (With Shiels.) Ginn.
201-206. Essays. *Mathematics Teacher*, 8:115; 9:77; *Science*, 44: 862, etc.
207-211. Reviews. *Bulletin Math. Soc.*, 22:192,463;23: 44, 136, etc.
212, 213. *Arithmetica Moderna*, Pri. and Adv. Spanish ed. Ginn.

1917 214-216. *Junior High Sch. Math.*, I, II, III. (With J. C. Brown.) Ginn.
217-224. *City Arithmetics*, 8 vols. Revision of 193-200.
225, 226. *Commercial Algebra*, I and II. (With W. S. Schlauch.) Ginn.
227. *Elementos de Algebra*. Spanish ed., Ginn.

228-236. Essays. *Bulletin Math. Soc.*, 23:266; 24:82; *T. C. Record*, etc.
237-239. Reviews. *Bulletin Math. Soc.*, 23: 176, 372, 375.

1918 240-243. Essays. *Mathematics Teacher*, 10: 179; etc.
244. *Union List of Math. Periodicals.* (With C. E. Seely.)
245-247. Reviews. *Science*, 47:144; *Bulletin Math. Soc.*, 24: 309, etc.

1919 248. *Higher Arithmetic.* Ginn.
249-251. *School Arithmetics*, I, II, II, Ginn.
252. *Number Stories of Long Ago*, Ginn.
253. Essay. *Mathematics Teacher*, 11:105.

1920 254-258. Essays. *Revista Math. Hispano-Am.*, 1920, p. 192, etc.
259-265. Reviews. *Amer. Math. Monthly*, 27:120, 129, 263, etc.

1921 266. *The Sumario Compendioso of Juan Diez*, Ginn.
267-289. Essays. *Amer. Math. Monthly*, 28:64, etc.
290-304. Reviews. *Ibid.*, 28:450; *Bulletin Math. Soc.*, 27: 479, etc.
305. *Computing Jetons.* Am. Numismatic Soc.

1922 306. *Fundamentals of Practical Math.* (With H. D. Harper.) Ginn.
307. *Machine Shop Mathematics.* (With H. D. Harper.) Ginn.
308. *Analytic Geometry.* (With L. P. Siceloff.) Ginn.
309. *Analytic Geometry, Brief Course.* See 308.
310. *Projective Geometry.* (With G. H. Ling.) Ginn.
311-323. Essays. *Amer. Math. Monthly*, 29:14, etc.
324-333. Reviews. *Mathematics Teacher*, 14: 407, etc.

1923 334-336. *Essentials of Geometry.* In three parts, Ginn.
337. *History of Mathematics*, Vol. I, Ginn.
338. *Mathematics.* (Greece and Rome Series.) Marshall Jones.
339. *Progress of Arithmetic in the last 25 years*, Ginn.
340-342. Essays. *T. C. Record*, 24:87; *Amer. Math. Monthly*, etc.
343, 344. Reviews. *Bulletin Math. Soc.*, 29:79, 274.

1924 345. *College Algebra.* (With L. P. Siceloff.) Ginn.
346. *Essentials of Algebra.* (With W. D. Reeve.)
347. *Essentials of Algebra.* New York edn. (See 346.)

348. *Essentials of Algebra, Complete Course.* (See 346.)
349. *Historical-Mathematical Paris.* Paris, privately printed.
350. Essay. *Isis*, 4 : 311.
351-353. Reviews. *Amer. Math. Monthly*, 31 : 252,299 ; etc.

1925 354. *Mathematica Gothica.* Paris, privately printed.
355. *History of Mathematics*, Vol. II, Ginn.
356-358. *New York State Arithmetics*, 3 vols. (With H. G. Burdge.) Ginn.
359. *Progress of Algebra in the last 25 years*, Ginn.
360. *Essentials of Algebra, Book II.* (See 346.)
361. *General H. S. Mathematics.* (With W. D. Reeve and J. A. Foberg.) Ginn.
362, 363. *Junior H. S. Mathematics*, I and II. (With J. C. Brown.) Ginn.
364. *The Geometry of René Descartes.* (Transl., with M. L. Latham.)
365-374. Reviews. *Amer. Math. Monthly*, 31 :43, 87, 135, 491, etc.
375-381. Essays. *Ibid.*, 32 : 287, 393, 444 ; *Mathematics Teacher*, etc.

1926 382-384. *Smith-Burdge Arithmetics*, I-III. Ginn.
385, 386. Reviews. *Mathematics Teacher*, 19 :57, 58.
387. Essay. *Amer. Math. Monthly*, 33 :28.
388-390. Essays. *Mathematics Teacher*, May number.
391. Essay. *Isis*, 8 :41.

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Eleanor Applegate, Audubon Junior High School, East 111th and Woodland Hills Park, Cleveland
South High School, Broadway, opp. Fullerton, Cleveland
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Thomas Jefferson Junior High School, West 46th Street and Oakley, Cleveland
Vera Baxter, Lincoln High School, Cleveland
A. C. Denison, 2950 West 14th Street, Cleveland
Lincoln High School Library, Cleveland
Edna M. Waite, Lincoln High School, Cleveland
Silisia M. Wright, 3104 Yorkshire Road, Cleveland Heights
Detroit Junior High School, Cleveland
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